Riemann Sums

Name______________________

A Riemann Sum is a calculation that uses rectanges to estimate the signed area *under* the graph of a function $f(x)$ between two lines $x = a$ and $x = b$.

R1. Calculate $R(4)$ the fourth Riemann Sum for the area under the graph of $f(x) = x^2 + 1$ between the values of 1 and 3 using right hand endpoints.

The procedure. First we find the rectangle width,

$$\Delta x = \frac{b-a}{4} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}.$$

We use Δx and the left endpoint $a = 1$ to locate the interval endpoints x_i, beginning with $x_0 = a$, that is $x_0 = 1$ in this case.

The next endpoint is called x_1 and $x_1 = x_0 + \Delta x$. So

$$x_1 = x_0 + \Delta x = 1 + \frac{1}{2} = 1.5$$

The next endpoint is called x_2 and $x_2 = x_1 + \Delta x$. So

$$x_2 = x_1 + \Delta x = 1.5 + \frac{1}{2} = 2$$

And so forth.

Fill in the x_i row on the table.

i	0	1	2	3	4
x_i	1	1.5			
$h_i = f(x_i)$	$1^2 + 1$				
h_i	2				
$A_i = f(x_i)\Delta x$	$2 \cdot \frac{1}{2}$				
A_i	1				

2. Plug each endpoint value x_i into the function $f(x) = x^2 + 1$.
3. Calculate the rectangle area $A_i = h_i \times \Delta x$
4. Add up the areas $R(4) = A_1 + A_2 + A_3 + A_4$.

R2. Calculate $R(2)$ the second Riemann Sum for the area under the graph of $f(x) = x^3 + x$ between the values of 0 and 2 using left hand endpoints.

$\Delta x =$

i	0	1	2	3	4
x_i	0				
$h_i = f(x_i)$					
h_i					
$A_i = f(x_i)\Delta x$					
A_i					

R3. Calculate $R(2)$ the second Riemann Sum for the area under the graph of $f(x) = x^3 + x$ between the values of 0 and 2 using left hand endpoints.