
CAKES

A set of anomalous cakes are shown to replicate themselves daily, and are to be modelled by $P(t) = 12e^{kt}$ where on the first day of their discovery there were 12 cakes. After 4 days, there are 68 cakes.

a. Find the value of k; write down the model.

b. How many cakes will there be after 1 week? After 10 days?

c. The cakes are stored in a space that will only hold 500 cakes. How long will it be until there is no more space?

d. How much faster are the cakes growing on day 10 than on day 4?

Answers.

a. $k = 0.4336503$ The cake count (population) is modelled by $P(t) = 12\,e^{0.43365\,t}$. where t denotes the time in hours since the snow began falling.

b. $P(7) = 250$ cakes $P(10) = 917$ cakes (rounded to whole numbers)

c. The time is $T(500) = 8.60$ hrs.

d. $P'(4) = 29.5$ in/hr $P'(5) = 397.8$ in/hr

e. 13.5 times faster.

CANCER CELLS

The number of cancer cells in a *growth cell* [1] is increasing exponentially. After 4 hours there are 50 cells. After 8 hours there are 300 cells.

a. Find the values of k and P_0 for the population model, then write down the model.

b. Use the model to predict how many cells there will be after 12 hours. After 20 hours? Is the model reasonable?

c. When does the model predict there will be 2,000,000 cells?

d. At what rate are the cancer cells increasing after 4 hours? After 8 hours?

e. How much faster are the cells growing after 8 hours, than after 4 hours?

Answers.

We will take t to be the time (in hours) where $t = 0$ will correspond to Hour 4. So at Hour 8, $t = 4$.

a. $k = 0.4479399$ The cancer cell population (the number of cells) is modelled by $P(t) = 50\,e^{0.44794t}$. where t denotes the time in hours since Hour 4.

b. $P(8) = 1800$ cells $P(16) = 64800$

c. The time is $T(2,000,000) = 23.40$ hrs.

d. $P'(0) = 22.4$ in/hr $P'(4) = 134.4$ in/hr

e. 6 times faster.

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[1] By a growth cell we mean a rectangular container in which cell growth is studied.

ELEPHANTS

The average weight of an elephant (in pounds) in an environment can be modeled by $W = 22.8\,e^{kt}$, where t represents the year with $t = 0$ corresponding to when the elephant was born. When the elephant is five years old, its weight is (about) 132 lbs.

a. Find the value of k; write down the model.

b. Use the model to predict the elephant's weight when it is 10 years old. And at age 15?

c. According to the model, at what age will the elephant weigh 340 pounds?

d. At what rate is the weight of the elephant increasing when it is seven years old? And 15 years old?

e. How much faster is the elephant's weight increasing when it is 15 years old than when it is 7 years old?

Answers.

a. $k = 0.351208$ The model for the elephant's weight is $W(t) = 22.8\,e^{0.3512t}$ where t is the number of years since birth.

b. $W(10) = 764.2$ lbs $W(15) = 4,424.4$ lbs

c. $T(340) = 7.7$ years.

d. $W'(7) = 93.6$ lbs/year $W'(15) = 1,553.9$ lbs/year

e. 16.60 times faster.

POPULATION

The population of China is increasing [2] and here we construct a *law of exponential growth* model for China's population. We take the initial population to be 250,000. We hypothesize that after 17 years the population is 5.2 times more than the population after (at) 2.5 years.

a. Find the value of k and P_0 for the population model.

b. Use the model to predict the population after 10 years.

c. According to the model, when will the population reach 1 million?

d. At what rate is the population increasing after 3 years? And after 6 years?

e. How much faster is the population increasing after 6 years than after 3 years?

Answers.

We will take t to be the number of years since the population was 250,000, and we will count people in thousands, so that 250 represents a population of 250,000.

a. $k = 0.1137006$ The population is modelled by $P(t) = 250\, e^{0.1137\, t}$. where t denotes the year.

b. $P(10) = 779.4$ thousand people.

c. The time is $T(1,000) = 12.19$ years.

d. $P'(3) = 40.0$ thousand people/year $P'(6) = 50.2$ thousand people/year

e. 1.26 times faster.

[2] Slowly, 0.6% according a Google publicdata page

SNOW

It is November 12, snowing, and the snow depth is growing exponentially. At 5:30 pm there are 2 inches of snow on the ground and after 2 hours there are 5 inches of snow.

a. Find the values of k and D_0 for the model; write down the model.

b. Use the model to predict how much snow there will be after 5 hours.

c. According to the model, when will the snow depth reach 9 inches?

d. At what rate is the population increasing after 3 hours? After 7 hours?

e. How much faster is the snow's height growing after 7 hours than after 3 hours?

Answers.

a. $k = 0.4581454$ The snow depth (in inches) is modelled by $D(t) = 2\,e^{0.45814t}$. where t denotes the time in hours since the snow began falling.

b. $D(5) = 19.76$ in

c. The time is $T(9) = 3.28$ hrs.

d. $D'(3) = 3.62$ in/hr $D'(7) = 22.64$ in/hr

e. 6.25 times faster.

ZOMBIE BACTERIA

The bacteria become infected when bitten by a zombie. The infected population (in thousands) can be modeled by $P = 0.974\,e^{kt}$ where t is the time in minutes. After 5 minutes the zombie bacteria population is 137,000.

a. Find the value of k, is the bacteria increasing or decreasing? Explain.

b. Use the model to predict the zombie bacteria at 14 minutes and at 21 minutes. Is the model reasonable? Explain.

c. According to the model, during what minute will the zombie Bacteria reach 669,000?

d. At what rate is the zombie bacteria increasing at 3 minutes? In 17 minutes?

e. How much faster is the zombie bacteria growing at 17 minutes than at 6 minutes?

Answers.

a. $k = 0.989265$ $\quad P(t) = 22.8\,e^{0.9893t}$ (kB)
b. $P(14) = 1,007,886$ kB $\quad P(21) = 1.025 \text{ x } 10^9$ kB
c. T(669) = 6.603 minutes.
d. $P'(3) = 18.74$ kB/min $\quad P'(17) = 19,391,931$ kB/min
e. $1,034,791$ times faster.