Math 181 — Anti-differentiating Power Functions — April 20, 2004

Suppose we are given a function $f(x)$ and we find a function $F(x)$ such that the derivative $F'(x) = f(x)$. Then $F(x)$ is called the antiderivative of $f(x)$. For example, consider the two functions $f(x) = 4x^3$ and $F(x) = x^4$. $F(x) = x^4$ is the antiderivative of $f(x) = 4x^3$. Why? Only because

$$\boxed{F'(x) = f(x)}$$

Here are a bunch of examples. 1. $f(x) = x^6 \Rightarrow F(x) = \dfrac{x^7}{7}$

2. $f(x) = x^{11} \Rightarrow F(x) = \dfrac{x^{12}}{12}$

3. $f(x) = x^{103} \Rightarrow F(x) = \dfrac{x^{104}}{104}$

4. $f(x) = 4x^6 \Rightarrow F(x) = 4\dfrac{x^7}{7}$

5. $f(x) = 24x^{11} \Rightarrow F(x) = 24\dfrac{x^{12}}{12} = 2x^{12}$

6. $f(x) = 4x^3 + 24x^{11} \Rightarrow F(x) = 4\dfrac{x^4}{4} + 24\dfrac{x^{12}}{12} = x^4 + 2x^{12}$

Is the basic rule clear? It just undoes the differentiation rule:

$$\boxed{\text{Differentiation: } f(x) = k \cdot x^n \Rightarrow f'(x) = k \cdot nx^{n-1}.}$$

The anti-differentiation rule is:

$$\boxed{\textbf{Anti-Differentiation: } f(x) = k \cdot x^n \Rightarrow F(x) = k\frac{x^{n+1}}{n+1}}$$

Here are a couple trickier examples.

7. $f(x) = 3x^{-5} \Rightarrow F(x) = 3\dfrac{x^{-4}}{-4}$

8. $f(x) = 6x^{-17} \Rightarrow F(x) = 6\dfrac{x^{-16}}{-16} = 3\dfrac{x^{-16}}{-8}$

9. $f(x) = 4x^{1/3} \Rightarrow F(x) = 4\dfrac{x^{4/3}}{4/3} = 3x^{4/3}$

10. $f(x) = 4x^{-1/3} \Rightarrow F(x) = 4\dfrac{x^{2/3}}{2/3} = 6x^{2/3}$

11. $f(x) = 3\sqrt{x} \Rightarrow F(x) = 3\dfrac{x^{3/2}}{3/2} = 2x^{3/2}$

12. $f(x) = \dfrac{3}{\sqrt{x}} \Rightarrow F(x) = 3\dfrac{x^{1/2}}{1/2} = 6x^{1/2}$

In each of these cases the hallmark is that the derivataive of $F(x)$ equals $f(x)$.

$$\boxed{F'(x) = f(x)}$$